SPH3U: Area and Averages

Recorder: ____________________
Manager: ____________________
Speaker: ____________________
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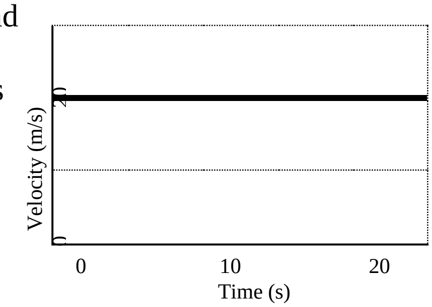
A graph is more than just a line or a curve. We will discover a very handy new property of graphs which has been right under our noses (and graphs) all this time!

A: Looking Under the Graph

A car drives along a straight road at 20 m/s. It is straight-forward to find the displacement of the car between 5 to 20 seconds. But instead, let's looks at the velocity-time graphs and find another way to represent this displacement.



1. Explain how to calculate the displacement of the car the familiar way.

Now let's map out that calculation visually **- on the graph.** There are two important quantities to represent – the velocity and the time interval.

2. Draw a horizontal line along the time axis of the graph that spans the time interval during which the car was moving.

3. Draw two vertical lines parallel to the velocity axis starting from the time axis and going to the line of the graph – this represents the velocity at the start and end moments (almost like a bar graph).

4. You have now constructed a shape which we can imagine located between the time axis and the line of the graph. What shape is formed? Shade in the shape on the graph. What characteristic of the shape could you easily calculate since you know the "length" of its two sides?

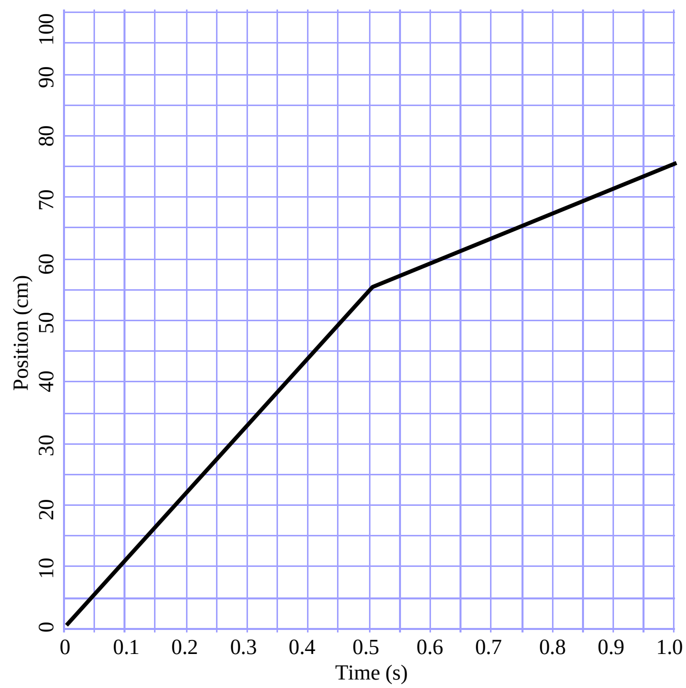
5. Perform this calculation for the area shaded in your graph. Be sure to show the proper physics units for each quantity used. What quantity does the final result represent?

You can imagine the line along the velocity axis travelling horizontally along the time interval and "sweeping out" an area corresponding to the car's displacement. In our example above, the larger the time interval, the greater the area that is swept out.

The area under a graph of velocity verses time for an interval of motion gives the displacement during that interval. Both velocity and displacement are vector quantities and can be positive or negative depending on their directions. According to our usual sign convention, areas above the time axis are positive and areas below the time axis are negative.

B: Kinky Graphs

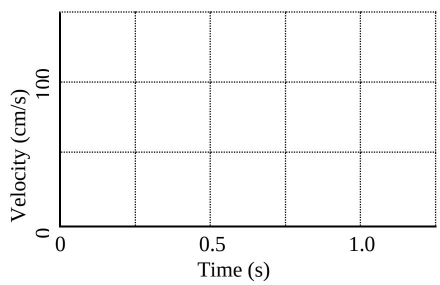
Here's a funny-looking graph. It has a kink or corner in it. What's happening here?



1. What characteristic of the graph is steady before the kink, steady after the kink, but changes right at the kink? What has happened to the motion of the object?

At exactly t = 0.5 seconds we cannot tell what the slope is – it is experiencing an abrupt change.

2. Sketch a velocity-time graph for this same motion.



To indicate a sudden change on a physics graph like the one above, use a dashed vertical line. This indicates that you understand there is a sudden change, but you also understand that you cannot have a truly vertical line.

3. What would a vertical line segment on a v-t graph mean? Is this physically reasonable? Explain.

4. Explain how to calculate the area under the v-t graph above. Find this result. Explain how to use the position-time graph to confirm your result.

5. We can perform a new type of calculation by dividing the area we found by the time interval. Carry out this calculation and carefully show the units.

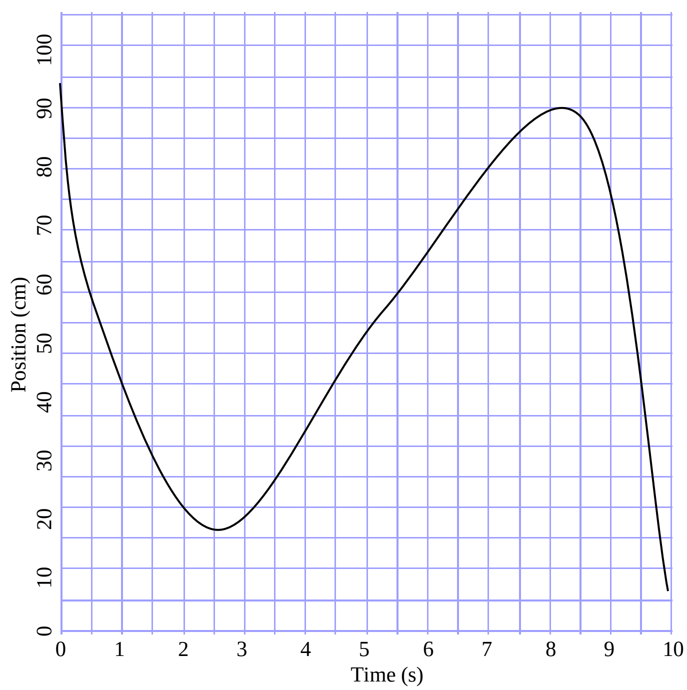
6. What type of velocity did you find from the previous calculation? How does it compare with the values in the v-t graph? Overall (during the entire time interval), is motion of the object uniform of nonuniform?

Part C: Average Velocity

Earlier in this unit, we noted that the ratio, $\Delta x/\Delta t$, has no simple interpretation if the motion of the object is nonuniform. Since the velocity is changing during the time interval, this ratio gives an *average velocity* for that time interval. One way to think about it is this: $\Delta x/\Delta t$ is the velocity the object *would* have *if* it moved with uniform motion through the same displacement in the same amount of time.

1. Use the interpretation above to help you draw a single line (uniform motion) on the d-t graph above and show that its slope equals the average velocity for that time interval. Show your work on the graph.

2. Use the same process to find the average velocity of the object in the graph to the right from t = 2 to t = 7 seconds. Show your work below.



3. Explain how, in two simple steps, you would find the average velocity for any interval on any position-time graph.

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